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Electronic polarizability of superconductors and inertial mass of a moving vortex

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Abstract. The problem of a vortex electromagnetic mass in a superconductor is considered accounting for the self-interaction effect conditioned by the coupling of the moving vortex to the excited fluctuations of the superfluid density. The obtained polaron-type mass exceeds the earlier obtained electromagnetic mass in view of the large value of the light speed relation to the Fermi velocity and can dominate over the vortex core mass.

PACS. 74.20.De Phenomenological theories (two-fluid, Ginzburg-Landau, etc.) – 74.25.Qt Vortex lattices, flux pinning, flux creep

Motion of Abrikosov vortices has a crucial impact on basic characteristics of superconductors, therefore the different regimes of vortex lattice dynamics has been studied for conventional and high-temperature superconductors. In the underdamped regime, when the equation of motion for the vortex cannot be reduced to the balance condition for external and damping forces, an important dynamical property is the inertial mass of the moving vortex. The discussion of effects arising due to the vortex mass can be found e.g. in references [1,2]. The inertial mass is an effective parameter that characterizes the increase of the internal energy of the superfluid when the vortex moves with a velocity $\mathbf{v}$, owing to origination of the kinetic energy $F_{kin} = Mv^2/2$. Various sources of a vortex kinetic energy in superconductors were proposed, such as the energy of the induced electromagnetic fields [2–6], the core energy [2,3,5–9], the energy of the hydrodynamic backflow [10], and the crystal lattice deformation energy [11–13].

Generation of the electromagnetic mass is the result of the polarization of the charged superfluid around the vortex core. In the region far from the core the modulus of the superconducting order parameter is roughly constant and the vortex motion causes only fluctuations of the order parameter phase, which implies excitation of the density fluctuations and the local polarization of the superfluid. In neutral compressible superfluids the density oscillations stimulated by a vortex motion create a significant "compressibility mass" [10,14,15], exceeding the core contribution. Meanwhile, in the charged liquids the density fluctuations are accompanied by the creation of electric fields which are strongly screened due to the Coulomb interaction. This screening reduces the energy of the density fluctuations. The electromagnetic mass of a vortex was first studied by Suhl [3] at the temperature $T = 0$ under the assumption of the perfect charge screening. In this work the induced electric field has been determined from the local charge neutrality condition, and the electric field energy was designated as the vortex kinetic energy. More accurate estimate of the electromagnetic mass at zero temperature has been made by Duan and Leggett [5] and Duan [6] with account of compressibility of the superfluid. In these works the local charge density and the electric field were computed by the self-consistent way using Maxwell's equations, while the vortex inertia was conditioned both by the electric field and the charge density oscillation energies. Nevertheless, the electromagnetic mass obtained by Suhl [3] coincides with the results of the papers [5,6] due to the fact that the charge screening length in superconductors is much smaller than the correlation length ξ. The electromagnetic mass found in the abovementioned works is exceeded by the vortex core mass owing to the smallness of the Fermi velocity v_F with respect to the light speed c: $M_{el}/M_{core} \propto (\lambda_L/\xi)^2 (v_F/c)^2$, where λ_L is the London penetration length.

Although in the papers [5,6] the charged superfluid compressibility was taken into account which permitted the correct description of the excited density oscillations, the role of vortex interaction with these excitations must be clarified for the proper determination of the vortex electromagnetic mass. A model of the vortex coupling to the low lying excitations of a neutral superfluid was proposed by Niu, Ao and Thouless [16]. We show in this paper that in superconductors the interaction of a vortex with dynamical polarization of the background permits an

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obvious description in the framework of the classical electrodynamics. This interaction generates a large electromagnetic mass, that can exceed the vortex core mass. The problem is discussed for the temperature region near absolute zero, when the concentration of normal electrons and the dissipation outside the vortex core vanish.

A vortex in a superfluid is the topological object that can be described by the phase χ of the order parameter $\psi = \Delta \exp\left(\int_L \nabla\chi d\mathbf{l}\right)$. If L is a closed contour encircling the point $\mathbf{r} = 0$ in the (x, y) plane, which is the coordinate of the singularity line directed along the z - axis of the coordinate system, then the single-valuedness of the order parameter is expressed by the equation $\oint \nabla\chi d\mathbf{l} = 2\pi$, allowing one to identify the phase around the static vortex with the azimuthal angle $\theta = arctg\,(y/x)$. In the case when the vortex moves uniformly with a small velocity $\mathbf{v}$ (when the adiabatic approximation for the phase is valid [15]), the phase is determined as $\chi(\mathbf{r}, t) = \theta(\mathbf{r} - \mathbf{v}t)$. The space and time derivatives of this phase

$$\nabla\chi = \hat{\mathbf{e}}_\varphi \frac{1}{r}, \dot{\chi} = -\mathbf{v}\nabla\chi \tag{1}$$

are the sources of the fields and currents around the moving vortex.

Induction of magnetic and electric fields by a singularity in superconductors results directly from the gauge invariance: the phase derivatives (1) enter into the energy functional in the proper combinations with the vector potential $\mathbf{A}$ and the scalar potential φ. In the region far from the core, where the order parameter modulus is constant, the energy functional[3] can be written as follows:

$$F = F_0 + \int d\mathbf{r} \left\{ \gamma_0 \left(\dot{\chi} + \frac{2e}{\hbar}\varphi \right)^2 + \gamma \left(\nabla\chi - \frac{2e}{\hbar c}\mathbf{A} \right)^2 + \frac{\mathbf{B}^2 + \mathbf{E}^2}{8\pi} \right\}. \tag{2}$$

Here F_0 is the energy of the homogeneous superconductor, i.e. the computing origin of the vortex energy; $\mathbf{B}$ and $\mathbf{E}$ are the induced magnetic and electric fields respectively. The integration goes over the two-dimensional radius-vector $\mathbf{r}$, as the vortex unit length mass should be estimated. The functional (2) describes only the phase fluctuations, therefore the parameters γ_0 and γ are dependent on the order parameter amplitude and determine the charge density and the current. The coefficient γ_0 can be expressed through the charge screening distance λ_{scr}, and γ - through the London penetration length λ_L:

$$\gamma_0 = \frac{1}{8\pi c^2} \left(\frac{\phi_0}{2\pi\lambda_{scr}} \right)^2, \ \gamma = \frac{1}{8\pi} \left(\frac{\phi_0}{2\pi\lambda_L} \right)^2, \tag{3a}$$

where $\phi_0 = \pi\hbar c/e$ is the flux quantum. The ratio of these parameters gives the square of the characteristic velocity

$$s^2 = \gamma/\gamma_0. \tag{3b}$$

In the energy functional used by Suhl [3] and in the further works [2,5,6] this velocity was assumed to be equal $s = v_F/\sqrt{3}$, then λ_{scr} represents the Fermi-Thomas screening length. Here, analyzing the problem in the framework of the phenomenological theory, we will not involve as yet the microscopic values of the parameters γ_0 and γ, preferring to use the phenomenological parameters s and λ_L determined by the equations (3a) and (3b).

The external magnetic field that has penetrated into the superconductor and created the topological defect can be described by a vector potential defined as $\mathbf{A}^{ext} = -(\phi_0/2\pi)\nabla\chi$. The induced magnetic field and the vortex currents which screen the external field are described by the potential $\mathbf{A} \equiv \mathbf{A}^{ind}$, so that the magnetic field $\mathbf{B}$ in (2) is $\mathbf{B} = \nabla\times\mathbf{A}^{ind}$. In a similar manner, we can define the external electric field $\varphi^{ext} = (\phi_0/2\pi c)\,\dot{\chi}$ which is screened by the induced charge density and creates the scalar potential φ^{ind}. As the slow motion of the vortex keeps the phase gradient (1) purely transverse, the induced vector-potential $\mathbf{A}^{ind}$ also have no longitudinal component (due to the gauge coupling of the order parameter to the electromagnetic field) and its time derivative determines the transverse field $-\partial_t\mathbf{A}^{ind}/c$. The electric field in (2) represents the sum of the transverse and longitudinal components: $\mathbf{E} = -\partial_t\mathbf{A}^{ind}/c - \nabla\varphi^{ind}$.

Let us now analyze in detail the origination of the longitudinal electric field around the moving vortex. The total electric field represents the sum of the induced scalar potential gradient and of the electric field generated by the moving magnetic field $\mathbf{B}$:

$$\mathbf{E}^{total} = -\nabla\varphi^{ind} - \frac{1}{c}[\mathbf{v}\times\mathbf{B}].$$

This expression can be obtained from the hydrodynamic equation for the charged superfluid flow in the presence of a magnetic field [10]. Essentially, the second term in the right-hand side of this expression contains the longitudinal component $[\mathbf{v}\times\mathbf{B}]^l/c$. This component can be revealed using $\mathbf{B} = \nabla\times\mathbf{A}^{ind}$ and the vector transformation $[\mathbf{v}\times\mathbf{B}] = -(\mathbf{v}\nabla)\mathbf{A}^{ind} + \nabla\left(\mathbf{v}\mathbf{A}^{ind}\right)$. Here the first term is equal to $\partial_t\mathbf{A}^{ind}$ and is responsible for the transverse electric field induction, while the second term creates a longitudinal field along with the scalar potential gradient. Thus the total electric field is equal to

$$\mathbf{E}^{total} = -\frac{1}{c}\frac{\partial\mathbf{A}^{ind}}{\partial t} - \nabla\left(\varphi^{ind} + \frac{1}{c}\mathbf{v}\mathbf{A}^{ind}\right).$$

The obtained longitudinal electric field is due to the transformation of the scalar potential in the laboratory frame (connected with the superconductor) which is the reference frame for the magnetic field motion:

$$\varphi = \varphi^{ind} + \frac{1}{c}\mathbf{v}\mathbf{A}^{ind} \tag{4}$$

The scalar potential (4) is just the one that must compose the gauge-invariant combination with the phase time-derivative that enters into the functional (2). Usually in this functional the scalar potential is assumed to be equal to φ^{ind}, omitting the second component in (4). Meanwhile, the account of this component of the scalar potential allows one to obtain the longitudinal current and the complete charge density in the Maxwell's equations, which must be derived by minimizing the functional (2) with respect to $\mathbf{A}^{ind}$ and φ^{ind}:

$$\nabla\times\mathbf{B}=\frac{4\pi}{c}\left(\mathbf{j}^l+\mathbf{j}^t\right)+\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{5a}$$

$$\nabla\mathbf{E}=4\pi\rho \tag{5b}$$

where the transverse supercurrent $\mathbf{j}^t$ and the charge density ρ with account of (3a) and (3b) can be written as

$$\mathbf{j}^t=-\frac{c}{4\pi}\lambda_L^{-2}\left(\mathbf{A}^{ind}-\frac{\phi_0}{2\pi}\nabla\chi\right) \tag{6a}$$

$$\rho=-\frac{c^2\lambda_L^{-2}}{4\pi s^2}\left(\varphi^{ind}+\frac{1}{c}\mathbf{v}\mathbf{A}^{ind}+\frac{\phi_0}{2\pi c}\dot{\chi}\right) \tag{6b}$$

and the longitudinal current $\mathbf{j}^l=\mathbf{v}\rho$ satisfies the continuity equation $\nabla\mathbf{j}+\partial_t\rho=0$. Using the equations (6a), (6b), and introducing the gauge-invariant potentials

$$\mathbf{Q}=\mathbf{A}^{ind}-\frac{\phi_0}{2\pi}\nabla\chi,\quad \Phi=\varphi^{ind}+\frac{1}{c}\mathbf{v}\mathbf{A}^{ind}+\frac{\phi_0}{2\pi c}\dot{\chi}$$

we can transform the functional (2):

$$F-F_0=\int d\mathbf{r}\left(-\frac{1}{2c}\mathbf{j}^t\mathbf{Q}+\frac{\mathbf{B}^2}{8\pi}\right)+\int d\mathbf{r}\left(-\frac{1}{2}\rho\Phi+\frac{\mathbf{E}^2}{8\pi}\right). \tag{7}$$

Here the first term in the right-hand side is the static vortex energy. The second term describes the kinetic energy due to the superfluid polarization. The kinetic energy must be completed with account of the energy of the longitudinal field $[\mathbf{v}\times\mathbf{B}]^l/c$. Besides, our aim is to include the effect of the vortex coupling to the superfluid polarization. Recalling that the total energy of a medium influenced by external fields contains the interaction energies of these fields with the magnetization $\mathbf{M}$ and the polarization $\mathbf{P}$ of the medium, we introduce now the vectors $\mathbf{M}$ and $\mathbf{P}$ for the charged superfluid surrounding the moving vortex, which are defined by the magnetization current $\mathbf{j}^m=-(c/4\pi)\lambda_L^{-2}\mathbf{A}^{ind}$ and the polarization charge $\rho^p=-\left(c^2\lambda_L^{-2}/4\pi s^2\right)\left(\varphi^{ind}+\mathbf{v}\mathbf{A}^{ind}/c\right)$ by means of the equations $\mathbf{j}^m=c\,[\nabla\times\mathbf{M}]$ and $\rho^p=-\nabla\mathbf{P}$. As the vortex is considered as the source of the external fields $\mathbf{A}^{ext}$ and φ^{ext}, the coupling of the vortex to the excitations of the superfluid can be clearly determined now: the interaction energy of the magnetized and polarized medium with the external fields is given by the expressions [17]

$$F_m=\frac{1}{2}\int\mathbf{M}\tilde{\mathbf{H}}d\mathbf{r} \tag{8}$$

$$F_p=\frac{1}{2}\int\mathbf{P}\tilde{\mathbf{E}}d\mathbf{r}. \tag{9}$$

Here $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{E}}$ are the fields which will remain in the superconductor if the magnetization and polarization vanishes, i.e. $\tilde{\mathbf{H}}=\nabla\times\mathbf{A}^{ext}$, and $\tilde{\mathbf{E}}=-\nabla\varphi^{ext}$ is the longitudinal component of the "external" electric field.

The interaction energies (8) and (9) must be added to the expression (7) to obtain the total energy of the superconductor. For the computation of the vortex energy the difference between origins of the energies F_m and F_p should be noticed. The field $\tilde{\mathbf{H}}=-(\phi_0/2\pi)\,curl\nabla\chi$ arises from the external field that penetrates as the vortex line into the superconductor and creates the current $\mathbf{j}^{ext}$. This field averaging over an array of vortices with the coordinates $\mathbf{r}_n$ with account of the relation $curl\nabla\chi=\hat{e}_z\cdot 2\pi\delta(\mathbf{r}-\mathbf{r}_n)$ results in $\left\langle\tilde{\mathbf{H}}\right\rangle=\hat{e}_z\cdot n\phi_0$, i.e. gives the externally penetrated flux density. So, the term F_m is connected with the external field energy and does not enter into the vortex static energy (given by the first integral in the equation (7)), which can be transformed making use of equation (5a) as

$$F_{st}=\frac{1}{2c}\int\mathbf{j}^{ext}\mathbf{Q}d\mathbf{r}. \tag{10}$$

On the contrary, electric fields are not imposed to the superconductor externally, and the field φ^{ext} is generated by the vortex motion, just as φ^{ind}. Therefore, the energy F_p (9) is the measure of the vortex inertia along with the second term in (7), and the complete kinetic energy is the sum of these two terms. This sum can be transformed to a more cogitable form by means of the electric field induction $\mathbf{D}=\mathbf{E}+4\pi\mathbf{P}$, which allows rewriting the equation (5b) as $\nabla\mathbf{D}=4\pi\rho^{ext}$, where the external charge density is $\rho^{ext}=-\left(c^2\lambda_L^{-2}/4\pi s^2\right)\varphi^{ext}$. According to the theory of a flowing conducting medium in a magnetic field [17], the longitudinal component of the induction vector is equal to

$$\mathbf{D}^l=\varepsilon\mathbf{E}^l+\frac{\varepsilon-1}{c}\left[\mathbf{v}\times\mathbf{B}\right]^l, \tag{11}$$

where ε is the longitudinal dielectric response function of the condensate. After some manipulations the complete electromagnetic kinetic energy is converted to the following form

$$F_{kin}=\int d\mathbf{r}\left(\frac{\mathbf{D}^l\mathbf{E}^l}{8\pi}-\frac{1}{2}\rho^{ext}\Phi\right). \tag{12}$$

The last expression does not include the energy of the induced transverse field which is negligible due to the large value of c.

The computation of the static and kinetic energies is straightforward with the use of Fourier transformations, such as $\nabla\chi=\int d\mathbf{k}e^{i\mathbf{kr}}(\nabla\chi)_k$, so that $(\nabla\chi)_k=\hat{\mathbf{e}}_\varphi\cdot i/2\pi k$ and $\dot{\chi}_k=-\mathbf{v}(\nabla\chi)_k$. To obtain the induced potentials from the Maxwell's equations (5a, 5b), it is convenient to introduce the dielectric response function

$$\varepsilon_k=1+\frac{c^2\lambda_L^{-2}}{s^2k^2}. \tag{13}$$

This function, that describes the condensate response to a longitudinal perturbation [18], possesses here only the space (but not the time) dispersion, due to the vortex slow motion $v\ll s$ resulting in creation of the "quasi-static" longitudinal electric field. The dielectric function

(13) allows one to write out the electric induction $\mathbf{D}$ in the form (8). Neglecting the displacement current in the equation (5a), we obtain two terms in the kinetic energy (12):

$$\frac{\mathbf{D}_k^l \mathbf{E}_k^l}{8\pi} = \frac{1}{8\pi s^2}\left(\frac{\phi_0}{2\pi}\right)^2 \frac{\varepsilon_k - 1}{\varepsilon_k} \frac{\dot{\chi}_k^2}{k^{-2} + \lambda_L^2}$$

$$-\frac{1}{2}\rho_k^{ext}\Phi_k = \frac{1}{8\pi s^2}\left(\frac{\phi_0}{2\pi}\right)^2 \frac{1}{\varepsilon_k} \frac{\dot{\chi}_k^2}{k^{-2} + \lambda_L^2}.$$

Here the second term gives the kinetic energy that has been calculated in the works [2–6]. The smallness of this energy is due to the large ε_k values in the wave vector range $k < \xi^{-1}$ where the used model is valid. The first term that results actually from the energy (9), at these wave vectors turns out to be larger than the second one provided that $c^2\lambda^{-2}/s^2k^2 \gg 1$. Notably, the sum of these two terms yields the exact expression for the kinetic energy

$$F_{kin} = \frac{1}{8\pi s^2}\left(\frac{\phi_0}{2\pi}\right)^2 \int d\mathbf{k} \frac{\dot{\chi}_k^2}{k^{-2} + \lambda_L^2} = \frac{F_{st} v^2}{2s^2} \qquad (14)$$

where the static energy is obtained from (10):

$$F_{st} = \left(\frac{\phi_0}{4\pi\lambda_L}\right)^2 \ln\frac{\lambda_L}{\xi} \qquad (15)$$

and the vortex mass is

$$M = \left(\frac{\phi_0}{4\pi\lambda_L}\right)^2 \frac{1}{s^2} \ln\frac{\lambda_L}{\xi}. \qquad (16)$$

This mass is generated by the vortex interaction with the condensate polarization (which is caused itself by the vortex motion), and can be regarded as a polaron-like mass. In the neutral systems the coupling of a vortex to the superfluid excitations gives rise to the vortex localization [16,19]. In superconductors the vortex self-interaction results in the effective mass increase, which is the feature of the polaron effect. A probable manifestation of the self-interaction can be also an attraction between fluctuating vortices, arising along with Van der Waals forces [20,21].

The vortex mass (16) is parameterized in fact by the parameter λ_{scr} that agrees with the original result of Suhl [3] (the equation preceding Eq. (3) of his paper). Suhl had derived the electromagnetic mass from the electric field energy $\int \left(E^2/8\pi\right) d\mathbf{r}$ and obtained the result

$$\mu_{em} = \frac{1}{2}\left(\frac{\phi_0}{4\pi\lambda_L}\right)^2 \frac{1}{c^2}\left(\frac{\lambda_L}{\lambda_{scr}}\right)^2,$$

which coincides with logarithmic accuracy and with account of the numerical factor 1/2 with our mass (16), since $\lambda_{scr} = (s/c)\,\lambda_L$. Then he argued that the inequality $\lambda_{scr} << \xi$ already holds in superconductors and the charge screening length must be substituted by the coherence length:

$$\mu_{em} = \frac{1}{2}\left(\frac{\phi_0}{4\pi\xi}\right)^2 \frac{1}{c^2}.$$

This result can be obtained from the second term on the right-hand side of equation (7), which indicates that the energy of the electric field coupled to the superfluid decreases due to non-complete charge density screening [4–6]. A rough order-of-magnitude estimate results in a conclusion, that such reduction of the kinetic energy is compensated by the account of the self-interaction energy (9). However, the basic distinction of our result from Suhl's original result should be noted. The equation (16) is in complete agreement with an important observation made in the works [4–6,8,12]: the contributions to the mass from various processes can be represented as $M_i = F_i/s_i^2$, where F_i is the corresponding static energy and s_i is the characteristic velocity. In contrast to the previous works, our analysis resulted *precisely* in the vortex mass $M = F_{st}/s^2$, where the static energy (15) is provided by the magnetic field and currents existing outside the vortex core and is described by the lower critical field H_{c1}: $F_{st} = (\phi_0/4\pi)\,H_{c1}$ [23]. As to the characteristic velocity, the mass (16) is determined by the velocity of the longitudinal electric field coupled to the condensate s, but not by the light speed c, that leads to the negligible mass. If we assume, as in [3], that this is the same speed $s = v_F/\sqrt{3}$ that determines the core mass $\mu_{core} = 3\xi^2 H_c^2/4v_F^2$ (where H_c is the thermodynamic field), then the mass (16) exceeds the core mass due to the factor $2\ln(\lambda_L/\xi)$ which can be quite large. The similar result has been obtained for the compressibility mass in the neutral superfluids [10, 14,15]. However, for the numerical estimate of the vortex mass some consideration of the velocity s is relevant here.

The energy functional (2) must describe oscillations of the order parameter phase, and in neutral Fermi superfluids these collective oscillations represent the Anderson-Bogolubov sound-like modes propagating with the velocity $v_F/\sqrt{3}$. For neutral Bose superfluids the sound velocity can be much smaller ($s \sim \hbar/m\xi$) that results in the large compressibility mass [10]. In superconductors the collective modes with the acoustic dispersion law are known to exist only at temperatures near the phase transition point. These are the Carlson - Goldman modes caused by the counterflow of normal and superfluid components of the charged superfluid (see Ref.[22] for a review). At zero temperature the Coulomb interaction prevents the propagation of acoustic modes in a charged superfluid; however, an "external" impact produced by the vortex motion causes compression of the superfluid. Therefore, the velocity s in this case should be considered as a parameter describing the electronic polarizability of the superconductor and determining the charge density according to equation (6b). The microscopic theory of longitudinal response of superconductors [22] allows determination of the required parameters at arbitrary temperatures. Particularly, in the dirty limit $l < \xi$, where l is the mean free path of electrons, at temperatures near T_c the Carlson-Goldman modes velocity $s_{CG} = \sqrt{2/3\pi} v_F l \xi^{-1}$ had been obtained, while at $T = 0$ one can find $s = v_F l \xi^{-1}/\sqrt{3}$, and the velocity s can be appreciably smaller than $v_F/\sqrt{3}$. Already at the impurity concentration corresponding to $l\xi^{-1} \sim 0.5$ and for Ginzburg-Landau parameter $\lambda_L/\xi \sim 10$ the mass (16) value exceeds more that ten times the core mass estimated in [3] (in this case the mass (16) for conventional low temperature superconductors is $M \sim 10^5\, m_e/cm$).

The experimental estimate of the vortex mass may be obtained by the study of microwave response of superconductors in the mixed state [25,26], determining the characteristic frequencies $\sqrt{\kappa/M}$ and $\sqrt{\eta/M}$ (where κ is the elastic force constant and η is the damping coefficient). The predicted mass value denotes that the resonance frequency must be three times less than that estimated using the core mass [3]. It should be noticed, however, that verification of the vortex mass value with the indicated accuracy requires also the precise determination of the elastic and viscous constants.

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